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Robust Design: An Overview

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Robust design has been developed with the expectation that an insensitive design can be obtained. That is, a product designed by robust design should be insensitive to external noises or tolerances. An insensitive design has more probability to obtain a target value, although there are uncertain noises. Theories of robust design have been developed by adopting the theories of other fields. Based on the theories, robust design can be classified into three methods: 1) the Taguchi method, 2) robust optimization, and 3) robust design with the axiomatic approach. Each method is reviewed and investigated. The methods are examined from a theoretical viewpoint and are discussed from an application viewpoint. The advantages and drawbacks of each method are discussed, and future directions for development are proposed.

Nomenclature

A_{cr}	=	area of common range
A_{sr}	=	area of system range
$\boldsymbol{b}$	=	design variable vector
$\boldsymbol{b}_L$	=	lower bound of $\boldsymbol{b}$
$\boldsymbol{b}_U$	=	upper bound of $\boldsymbol{b}$
DP_i	=	ith design parameter
d_i^-	=	shortage of the ith goal
d_j^+	=	overachievement of the ith goal
$\mathrm{d}(\boldsymbol{z}^b, \boldsymbol{z}^p)$	=	joint probability density function
$E[\cdot]$	=	expectation value of $\cdot$
$F(\boldsymbol{b}, \boldsymbol{p}, z)$	=	objective function with noises
FR_i	=	ith functional requirement
$f(\boldsymbol{b})$	=	objective function
f_{new}	=	normalized multiobjective function
$G_j(\boldsymbol{b}, \boldsymbol{p}, z)$, $g_{j,\mathrm{new}}$	=	jth constraint with noises
$g_j(\boldsymbol{b})$	=	jth inequality constraint
g_j^f	=	goal of the jth objective function
$h_i(\boldsymbol{b})$	=	ith equality constraint
$h_i(d_i^-, d_i^+)$	=	function made of d_i^- and d_j^+
I	=	information content
k	=	constant for the loss function, constant for $g_{j,\mathrm{new}}$
$L(f)$	=	loss function
m	=	target value of the characteristic function f
P	=	probability for satisfaction of a constraint
$P(\boldsymbol{b})$	=	penalty function
$P_{j,0}$	=	lower limit of P
p	=	probability of success
$\boldsymbol{p}$	=	design parameter vector
q	=	number of equality constraints
r	=	number of inequality constraints
s	=	scale factor
$u_i(z_i^b)$	=	probability density function of z_i^b
$v_i(z_i^p)$	=	probability density function of z_i^p
w_i	=	weighting factor for the ith objective function
$\boldsymbol{z}^b$	=	noise vector of $\boldsymbol{b}$
$\boldsymbol{z}^p$	=	noise vector of $\boldsymbol{p}$
η	=	signal-to-noise ratio
μ	=	mean of characteristic function f
$\boldsymbol{\mu}_b$	=	mean vector for $\boldsymbol{b}$ when the noises are considered
μ_{b_i}	=	mean of b_i
$\mu_f(\boldsymbol{b}, \boldsymbol{p})$, μ_f	=	mean of characteristic function f
μ_f^*	=	base value for μ
$\boldsymbol{\mu}_p$	=	mean vector for $\boldsymbol{p}$ when the noises are considered
σ	=	standard deviation of characteristic function f
σ_{b_i}	=	standard deviation of b_i
$\sigma_f(\boldsymbol{b}, \boldsymbol{p})$, σ_f	=	standard deviation of characteristic function f
σ_f^*	=	base value for σ
σ_{g_i}	=	standard deviation of g_i
σ_{p_i}	=	standard deviation of p_i
σ_y	=	standard deviation of characteristic function y
$\phi_j(p_j)$	=	variable for probability
$\psi(\boldsymbol{b})$	=	characteristic function of inner array

I. Introduction

AS the engineering environment becomes extremely competitive, quality products are required in industries. Unexpected deviations from the function that a designer initially intended for are caused by variations in various engineering processes. Robust design is to prevent such phenomena. Robust design has been developed to improve product quality and reliability in industrial engineering. Recently, this technology has been expanded to various design areas.

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First, we survey the definition of robust design. Many researchers have defined it. Taguchi, who is the pioneer of robust design, said "robustness is the state where the technology, product, or process performance is minimally sensitive to factors causing variability (either in the manufacturing or user's environment) and aging at the lowest unit manufacturing cost."[1] Suh said "robust design is defined as the design that satisfies the functional requirements even though the design parameters and the process variables have large tolerances for ease of manufacturing and assembly. This definition of robust design states that the information content is minimized."[2] Box said that "Robustifying a product is the process of defining its specifications to minimize the product's sensitivity to variation."[3] Although different expressions are used, their meanings are similar. A common aspect of the definitions is that robust design is a design insensitive to variations, and this concept is accepted in the engineering community.

Conventional design methodologies have been developed to improve the performances of products.[4–7] If consideration of robustness is added, a new direction should be taken in addition to the conventional concept. Theories of robust design have been developed by using existing design theories. The first one is the Taguchi method, and it is the foundation of robust design (see Refs. 8–10). Taguchi proposed methods of determining variables to make performance insensitive to noises in the manufacturing process. The concept of robustness was established by the Taguchi method. In a product design, robustness is obtained by appropriate modification of the Taguchi method. The second one is robust optimization, where the robustness concept is added to conventional optimization.[11–19] The objective and the constraint functions are redefined with robustness indices. The last one is axiomatic design, which can be exploited well in conceptual design. In axiomatic design, the independence axiom is utilized to find excellent designs. When multiple designs are found by using the independence axiom, the best one is selected by the information axiom, and the robustness concept is employed in this process.[2,20–22]

In this research, various robust design methodologies are overviewed based on the preceding three methods. The trend of the research is surveyed, and the application is discussed. However, theoretical aspects are emphasized more rather than application aspects. The advantages and disadvantages of the methods are also discussed. In the robust design area, some terminologies have been used in a different way. Thus, those terminologies are precisely defined, and a unified viewpoint of robust design is proposed.

There are some methods that are similar to robust design. They are design methods considering reliability and uncertainty.[23–25] Uncertainties or noises are included in these methods as well. Optimization terminologies can be utilized to distinguish these methods from robust design. In robust design, insensitiveness of the objective function is emphasized. In reliability design, reliability of constraints is important. The reliability of constraints is similar to the robustness of the constraints in robust design. It is calculated exactly by a probability theory in reliability design, whereas it is directly calculated in robust design. Design with uncertainties is similar to robust optimization. However, robustness of the objective function is not considered. The distinctions are made based on the methods of the early stages. These days, the methods are fused, therefore, it may not be possible to distinguish them in some applications. Methods considering reliability or uncertainties are not reviewed in this paper.

II. Mean and Variance

When the optimization framework is used, a design problem can be formulated as follows[26,27]: Find

$$\boldsymbol{b} \in R^n \tag{1}$$

to minimize

$$f(\boldsymbol{b}) \tag{2}$$

subject to

$$h_i(\boldsymbol{b}) = 0, \qquad i = 1, \ldots, q \tag{3}$$

$$g_j(\boldsymbol{b}) \leq 0, \qquad j = 1, \ldots, r \tag{4}$$

$$\boldsymbol{b}_L \leq \boldsymbol{b} \leq \boldsymbol{b}_U \tag{5}$$

The equality constraints may not be included in the formulation because they are usually eliminated before the optimization process.

The preceding functions can be evaluated by two methods. One is to use experiments when the function is not mathematically defined. A number of case studies using the traditional Taguchi method are attributed to this case. The other is to use mathematical expressions that are directly calculated or approximated by numerical approaches. The present review is mainly focused on the latter method. When noises or perturbations exist, the objective and the constraint functions are modified as follows:

$$f(\boldsymbol{b}) \rightarrow f(\boldsymbol{b} + \boldsymbol{z}^b, \boldsymbol{p} + \boldsymbol{z}^p) \tag{6}$$

$$g_j(\boldsymbol{b}) \rightarrow g_j(\boldsymbol{b} + \boldsymbol{z}^b, \boldsymbol{p} + \boldsymbol{z}^p) \tag{7}$$

where $\boldsymbol{p} \in R^0$ is the design parameter vector, and it is usually regarded as a constant vector in the design process. The noises of design variables and design parameters are represented as $\boldsymbol{z}^b \in R^n$ and $\boldsymbol{z}^p \in R^0$, respectively, where 0 is the number of parameters. The noise factors $\boldsymbol{z}^b$ and $\boldsymbol{z}^p$, are also called the uncontrollable factors, in the sense that they cannot be controlled by the designer.[1,8,9] The noise factor may be an external, an internal, or a unit-to-unit noise of a product. They are summarized in Table 1.[28,29]

The control factors are selected as independent variables that can be controlled by the designer and have significant impact on the problem characteristics. The control factors correspond to the design variable vector $\boldsymbol{b}$, whereas the characteristic function is equivalent to the objective function f of the optimization problem. The characteristic function is a function or response that is to be measured and improved on.

When Eqs. (6) and (7) are substituted into Eqs. (1–5) to consider the robustness in the design formulation, Eqs. (1–5) are rewritten as follows: Find

$$\boldsymbol{b} \in R^n \tag{8}$$

to minimize

$$F(\boldsymbol{b}, \boldsymbol{p}, \boldsymbol{z}) \tag{9}$$

subject to

$$G_j(\boldsymbol{b}, \boldsymbol{p}, \boldsymbol{z}) \leq 0, \qquad j = 1, \ldots, r \tag{10}$$

$$\boldsymbol{b}_L \leq \boldsymbol{b} \leq \boldsymbol{b}_U \tag{11}$$

where F and G_j are the functions considering the noises in f and g_j, respectively. Generally, F and G_j are defined as the functions of the mean and the variance of f and g_j, respectively.

Robust design should not be developed by the deterministic concept. It should be developed by the stochastic concept because the deterministic approach cannot include the noises. The performance

Table 1 Variation sources and examples

Factors	Examples
External	Temperature and relative humidity
	Load
	Human error
	Dust in the environment
Unit to unit	Weight of each part
	Dimension tolerance
	Differences by batch-to-batch
	Thickness variations in coated products
Internal (aging)	Wear
	Mileage of a car
	Weathering of paint on a house
	Total current passed through a car battery
	Plastic creep

Table 2 Formulas of means and variances

Name	Symbol	Formula[a]
Sample mean	$\bar{f}$ or (m_f)	$\bar{f} = \dfrac{1}{k}\sum_{i=1}^{k} f_i$
Sample variance	V or $\left(s_f^2\right)$	$V = \dfrac{1}{k-1}\sum_{i=1}^{k}(f_i - m_f)^2$
Sample standard deviation	s_f	$s_f = \sqrt{V}$
Population mean	μ	$\mu = \dfrac{1}{l}\sum_{i=1}^{l} f_i$
Population sample variance	σ^2	$\sigma^2 = \dfrac{1}{l}\sum_{i=1}^{l}(f_i - \mu)^2$
Population sample standard deviation	σ	$\sigma = \sqrt{\sigma^2}$

[a]Here k is the number of samples and l is the number of population.

measures are defined as the data to be measured experimentally or mathematically, which represent the performances of a product or a process. The performance measures are usually modified to form a characteristic function. The performance measures could vary under the same circumstance due to the distribution of noise factors. Generally, the distribution is measured by the mean and the variance. The formulas of mean and variance are summarized in Table 2, where f_i is the ith characteristic function in experiment or the ith response function in the design.[30]

When Eqs. (6) and (7) are considered, the mean μ_f and the variance σ_f^2 of the characteristic function f are represented as[10,31]

$$\mu_f = E[f(\boldsymbol{b})] = \int\int\cdots\int f(\boldsymbol{b}+\boldsymbol{z}^b, \boldsymbol{p}+\boldsymbol{z}^p)u_1\left(z_1^b\right)\cdots u_n\left(z_n^b\right) \times v_1\left(z_1^p\right)\cdots v_0\left(z_0^p\right)\mathrm{d}z_1^b\cdots \mathrm{d}z_n^b\,\mathrm{d}z_1^p\cdots \mathrm{d}z_0^p \tag{12}$$

$$\sigma_f^2 = E[f(\boldsymbol{b})-\mu_f]^2 = \int\int\cdots\int\left\{f(\boldsymbol{b}+\boldsymbol{z}^b, \boldsymbol{p}+\boldsymbol{z}^p)-\mu_f\right\}^2 \times u_1\left(z_1^b\right)\cdots u_n\left(z_n^b\right)v_1\left(z_1^p\right)\cdots v_0\left(z_0^p\right)\mathrm{d}z_1^b\cdots \mathrm{d}z_n^b\,\mathrm{d}z_1^p\cdots \mathrm{d}z_0^p \tag{13}$$

where $u_i(z_i^b)$ and $v_i(z_i^p)$ are the probability density functions of noise factors z_i^b and z_i^p, respectively. In Eqs. (12) and (13), the noise factors are assumed to be statistically independent. If it follows the Gaussian distribution, the probability density function becomes as[28]

$$u_i\left(z_i^b\right) = \left(1\big/\sigma_{b_i}\sqrt{2\pi}\right)\exp\left[-\left(b_i-\mu_{b_i}\right)^2\big/2\sigma_{b_i}^2\right] \tag{14}$$

where μ_{b_i} and σ_{b_i} are the mean and the standard deviation of the ith design variable b_i.

Robust design techniques have been developed by replacing the functions of the mean and variance into the functions of Eqs. (8–10). In the next sections, robust design methodologies are examined.

III. Taguchi Method

A. Taguchi Method

By the end of the 1940s, the Taguchi method had been developed by for quality improvement. His technique for quality engineering is referred to as the Taguchi method or robust design.[1,30] Whereas the Taguchi method was successfully applied at the Electrical Communications Laboratories of the Nippon Telephone and Telegraph Co., AT&T Bell Laboratories was interested in it. In 1980, Phadke[10] invited Taguchi to AT&T Bell Laboratories. After being impressed by the Taguchi method, Phadke published a textbook on the Taguchi method in 1989.[10]

The Taguchi method has greatly contributed to quality improvement of various designs. In early case studies, the Taguchi method was applied to the process design rather than the product design.

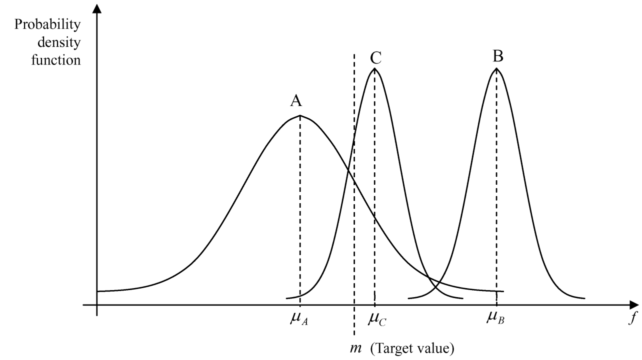

Fig. 1 Examples of distribution of the response functions.

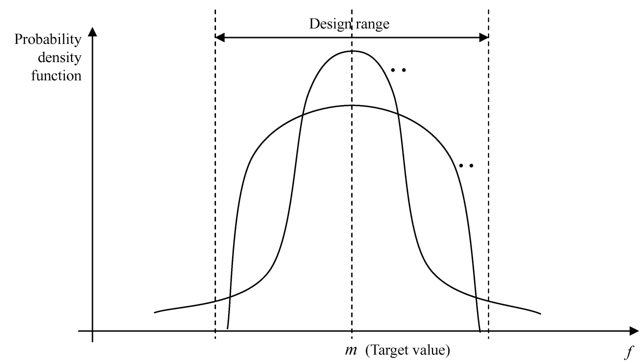

Fig. 2 Probability density functions of two designs with design range.

That is, it was regarded as a method for design of experiments rather than a design methodology.[8–10]

In Eqs. (8–11) of the design formulation, the original Taguchi method does not deal with the constraints in Eq. (10). It leads to unconstrained optimization problems. Although the design parameters $\boldsymbol{p}$ and the noise factors $\boldsymbol{z}$ are not explicitly defined in the design, the Taguchi method can be used to determine the settings of control factors for robust design. To explain this in detail, let us take an example with three distributions of the objective function f as shown in Fig. 1.

In Fig. 1, μ_A, μ_B, and μ_C are the means of distributions A, B, and C, respectively. Then, robustness can be measured by the magnitude of variation. When the design is focused only on the robustness of the performance, the candidate designs B and C are better than design A. However, the target value of the performance generally exists in the design. The target value can be zero, infinite, or a specific nominal value. Thus, the Taguchi method suggests a design with minimum variation around the target value. From the viewpoint of the Taguchi method, design C is the best out of the three candidate designs.

The procedure to obtain a robust design can be explained as follows: The design candidates D and E with two arbitrary distributions are represented in Fig. 2. The design range is equivalent to the allowable range, and the means of the two distributions coincide with the target value m of the design. A small part of candidate design D is outside the design range, whereas none of candidate design E is outside the design range. However, if any unexpected noise factors become active, the distribution of E has a larger probability to be outside the design range than that of D. For instance, the unexpected noises are the tolerances of product sizes and variances of the environment such as temperature, humidity, external forces, manufacturing process, etc. Thus, it is clear that candidate design D is more robust than candidate design E.

Taguchi introduced a quadratic loss function to represent robustness as (see Ref. 10)

$$L(f) = k(f-m)^2 \tag{15}$$

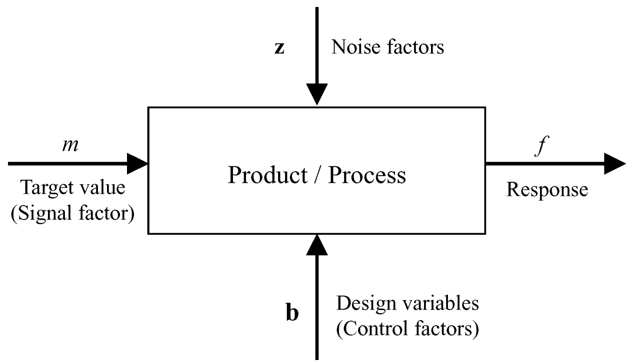

Fig. 3 Block diagram of a product/process: P diagram.

where k is the constant to define the loss and m is the target value. The expected value of the loss function is defined as[10]

$$Q = E[L(f)] = k[\sigma^2 + (\mu - m)^2] \quad (16)$$

where μ is the mean of f and σ is the standard deviation of f.

The symbol f represents the response or objective function in a general design, whereas the symbol y is used for f in the Taguchi method. Robust design is the design minimizing the average loss, which can select design candidate D in Fig. 2.

Figure 3 is called the P diagram and used to explain the procedure of the Taguchi method from another viewpoint. The P diagram represents the schematic relationship between the factors and the product or process. The noise factor is the one that the designer cannot control even though it causes variability. In a general design, tolerances are considered as noise factors. In a parameter design, control factors are determined to reduce the effect of the noise factors. Therefore, noise factors are not directly considered. On the other hand, noise factors are directly controlled in a tolerance design. The parameter design is the stage to improve the quality of a product or process without cost increase, whereas the tolerance design does with cost increase. In this paper, the parameter design is explained.

When the target value of a response is given as shown in Fig. 3, the Taguchi method determines the optimum setting of control factors so that the variation of a response is minimized, although uncontrollable factors (noise factor) exist. Equation (16) may be regarded as an index to find a robust design. Suppose that we have a scale factor s to adjust the current mean to the target value. The scale factor s is described as

$$s = m/\mu \quad (17)$$

When the current mean is adjusted to the target value, the average loss function of Eq. (16) is changed to[10]

$$Q_a = k[\{\mu(m/\mu) - m\}^2 + \{(m/\mu)\sigma\}^2] = km^2(\sigma^2/\mu^2) \quad (18)$$

To enhance additivity of the effect of the control factors, Eq. (18) is transformed to[10]

$$\eta = 10\log_{10}(\mu^2/\sigma^2) \quad (19)$$

Equation (19) is the ratio of the power of signal factors μ and the power of noise factors σ. Thus, it is called the signal-to-noise (S/N) ratio. Maximizing Eq. (19) is equivalent to minimizing Eq. (18). That is, a robust design is obtained by maximizing Eq. (19).

As in the example of Fig. 2, the response with target value m is referred as the nominal-the-best type characteristic. On the contrary, the responses with the target values of zero and infinity are referred as the smaller-the-better type and the larger-the-better type, respectively. The S/N ratios of the smaller-the-better type and the larger-the-better type can be derived by the similar procedure as for the nominal-the-best type. As the index of robustness, the S/N ratio of Eq. (19) is preferred to the average loss function of Eq. (16). The examples of S/N ratios are summarized in Table 3.[10,32]

Table 3 Examples of S/N ratios

Characteristic type	S/N ratios
Nominal the best	With scale factor $\eta = 10\log(\mu^2/\sigma^2)$ With adjustment factor $\eta = -10\log\sigma^2$
Smaller the better	$\eta = -10\log\left[\frac{1}{n}\sum_{i=1}^{n} y_i^2\right]$
Larger the better	$\eta = -10\log\left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right]$

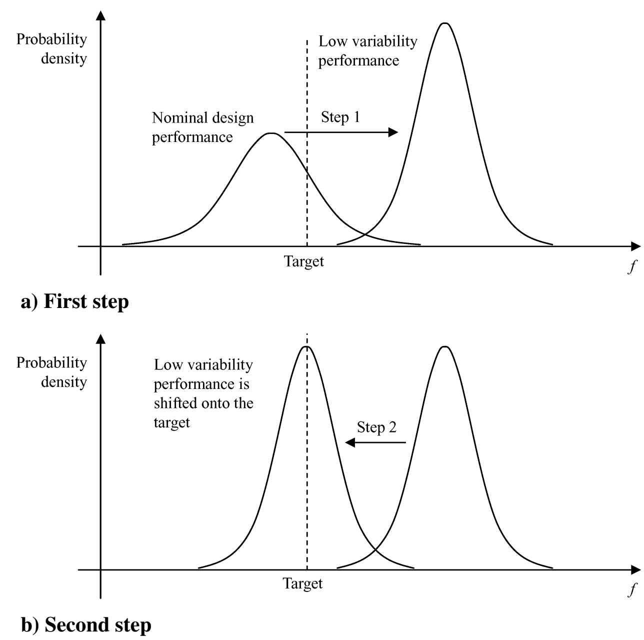

Fig. 4 Two steps of Taguchi method.

There are two goals in performing robust design. One is to minimize the variability produced by the noises factors. The other is to make the mean value close to the target value. The design to attain one goal is not usually consistent with the one to attain the other goal. To meet the two goals, Taguchi developed a two-step optimization strategy. The first step is to reduce the variation, and the second step is to adjust the mean on the target. The procedure is shown in Fig. 4. The first step, shown in Fig. 4a, concentrates on minimization of the variation, while the mean is ignored. In the second step (Fig. 4b), the mean is moved to the target value while sacrificing the improved variation somewhat.[1,8–10]

The first step is to find the optimum setting of control factors (design variables) to maximize the S/N ratio or to reduce the variation. The orthogonal arrays are employed to assign the control factors to an experimental matrix. The best condition can be selected from the full combinations of control factors. However, it is efficient to select the smallest orthogonal array because full combinations are costly in most cases. An orthogonal array is a simple experimental matrix to determine the factor effects by ignoring the high interactions between factors. Since Rao developed orthogonal arrays in 1946, they have been widely utilized in the field of statistics and design of experiments.[33]

The smallest size of an orthogonal array can be selected by using the number of control factors and the number of levels.[10] In a matrix experiment, experiments for a row of the orthogonal array called an inner array are repeatedly conducted to evaluate the effects of noise factors.[10,11,30,33] From the experiments for each row, the S/N ratios are calculated. Then, the optimum setting of control factors is determined by performing analysis of means (ANOM) of the S/N ratios. A one-way table is utilized in the ANOM process. If the average loss of Eq. (16) is defined as the characteristic function to find a robust design, the effect on the variation is confounded by the effect on the mean. Therefore, the S/N ratio adjusted to the target value of Eq. (19) or the S/N ratio defined as Eq. (20) has been utilized as the characteristic in the nominal-the-best type problem,

$$\eta = -10 \log_{10} \sigma^2 \tag{20}$$

When significant interactions exist, they should be considered in selecting the orthogonal array and assigning the factors. However, it is not easy to identify the significant interactions among the control factors in design. This can be a drawback of using orthogonal arrays. Thus, the optimum setting determined by performing ANOM should be followed by a confirmation experiment.

A computer simulation model provides the same output with the same input because it is a deterministic method. Although it is a disadvantage, the computer simulation model has an advantage in that it can define the design parameters $\boldsymbol{p}$ and the noise factors $\boldsymbol{z}$ exactly. In this case, the Taylor series expansion, the Monte Carlo simulation, or an outer array can be utilized to obtain the variations (see Ref. 10). An outer array is an orthogonal array, which is used for a row of the orthogonal array (inner array). The outer array has multiple cases of experiments for a row; therefore, it accommodates the repeated experiments of an experiment case.

The second step is to set the mean to the target value by selecting a control factor called the scale factor. The scale factor should only have an effect on the mean, while the S/N ratio is maintained. In general experiments, time can be one of the scale factors. However, it is difficult to find a scale factor in a general design. Furthermore, no scale factor may exist, and this prevents the designer from applying the Taguchi method directly to the design.

B. Research Topics of the Taguchi Method

The Taguchi method has been widely applied to various fields due to its simple and explicit procedures.[1,8–10,34–37] Even beginners can easily use the Taguchi method. Especially, the Taguchi method has been regarded as an excellent tool in a design process using experimental approaches. Based on the concepts of the traditional Taguchi method, researchers of design methodologies have tried to use Taguchi's outstanding ideas in their fields. However, there are some conflicts between the traditional Taguchi method and the design methodologies. Thus, research has been performed to overcome the difficulties.

First, many researchers[35,37–40] have pointed out the possibility of using the S/N ratio in an incorrect manner. Leon et al.[38] found the condition of the transfer function in which the S/N ratio of Eq. (19) is effective. It is that the S/N ratio could be a performance measure independent of adjustment (PerMIA) only when a scale factor exists and the standard deviation is proportional to the mean. Box[35] and Box et al.[39] suggested a more general transformation approach and proved that the variance of a transformed variable becomes a PerMIA. Their research is developed based on the assumption that a scale factor exists.

The performance criterion such as the S/N ratio is only effective when factors influencing the mean are separated from the factors influencing the variance. However, the effects on the mean confound with the effects on the variance when the factors influencing the mean cannot be separated. The same difficulties exist with the S/N ratios of the smaller-the-better and the larger-the-better type characteristics.[40] Thus, one should identify whether the S/N ratio can give a robust design before solving the problem.

Phadke[10] and Fowlkes and Creveling[30] tried to obtain the validity of the S/N ratio by examining the qualification of the characteristic function. They suggested some guidelines. However, the guidelines are quite strict, and it is difficult to determine whether they are satisfied. Montgomery[40] recommended the use of Eq. (20) instead of Eq. (19) as the S/N ratio, and Lee et al.[11] used the weighted multiobjective function for robust design. Wu[41] also evaluated the performance criteria of the S/N ratio, mean standard deviation, and variance.

Second, research has been performed to reduce the number of experiments in the traditional Taguchi method. The traditional method requires as many experiments as $N_{\text{in}} \times N_{\text{out}}$ (where N_{in} is the number of experiments in the inner array and N_{out} is the number of experiments in the outer array) times. This may result in an inefficient and costly process. Welch et al.,[42] Vining and Myers,[43] Shoemaker and Tsui,[44] and Kwon and Moon[45] indicated shortcomings and suggested the combined array technique. A few researchers[40–42] developed the approximate models of the mean and the variance based on the response surface models. However, the reliability of the robust design determined from the approximate models strongly depends on the reliability of the approximate models.

Third, research that deals with more than one characteristic function has been performed because the traditional Taguchi method provides robust design for a single characteristic function. Design problems with more than one characteristic function are often met. To obtain robust design in the problem with multiple characteristic functions, Phadke[10] determined the robust optimum by the designer's decision on tradeoffs. Shiau[46] and Tai et al.[47] introduced the weighting factors to unify each characteristic function. However, these methods strongly depend on the designer's intuition. Tong and Su[48] determined the weighting factors by using the fuzzy theory to avoid the described ambiguity.

Pignatiello[49] derived the loss function of multiple characteristic functions, and Tsui[50] suggested robust design by deriving the expectation value of Pignatiello's loss function. In addition, Wu[51] suggested the following procedure: 1) The optimum levels corresponding to each characteristic function are calculated. 2) The percentage reduction of quality loss is determined by recording the change in the S/N ratio for each optimum. 3) The optimum is determined as the levels with maximum percentage reduction and minimum approximated loss function. Recently, Liao and Chen[52] suggested the data envelopment analysis ranking to consider multiple characteristic functions. Similar to the optimization problem with a multiobjective function,[53] the method has both advantages and disadvantages. Thus, the designer should deliberately select an appropriate method for one's problem.

Finally, the Taguchi method cannot include the constraints of Eq. (10). To include the constraints and to obtain a robust design for the problem without the scale factor, Lee and Park[12] and Lee[31] suggested the following procedures. 1) The deterministic optimum is calculated. It is assumed that the target value is obtained in this process. 2) Near the optimum, the candidate design variables are assigned to an orthogonal array, which is called the inner array. 3) Each row of the inner array generates the outer array considering the noises, and the experiments are performed. 4) From the results of the outer array, the characteristic function of the inner array is determined as

$$\Psi(\boldsymbol{b}) = \sigma_y + P(\boldsymbol{b}) \tag{21}$$

where $P(\boldsymbol{b})$ is the penalty function that is the maximum violation of the experiments in the outer array. 5) The ANOM of the characteristic of Eq. (21) is performed to estimate the characteristic at an arbitrary design. 6) For each combination of the candidate design variables, the characteristic is estimated. 7) After the estimators are arranged by ascending order, the optimum is selected as the set with the lowest estimator and $P(\boldsymbol{b}) = 0$. However, this method selects a robust design with limited candidate designs. As already mentioned, recent research has been performed to supplement the deficiencies of the traditional Taguchi method.

IV. Robust Optimization

A. Robust Optimization

Aforementioned robust design problems have been solved based on well-developed mathematical optimization techniques during the 15 years.[42,43,54–56] Because the Taguchi method uses an orthogonal array and design variables are defined in a discrete space, it is difficult to treat a wide design range. Design in a discrete space may have an advantage; however, design in a continuous space is often required as well. Generally, a design may have many design constraints that the Taguchi method may not resolve. Because optimization techniques can easily handle the constraints, robust optimization based on optimization techniques has been studied.[57] In a general

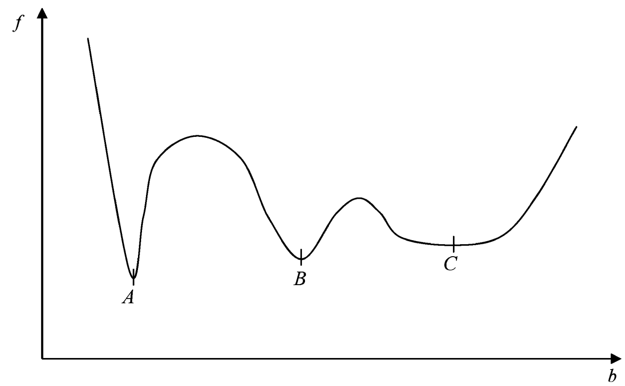

Fig. 5 Robustness of the objective function.

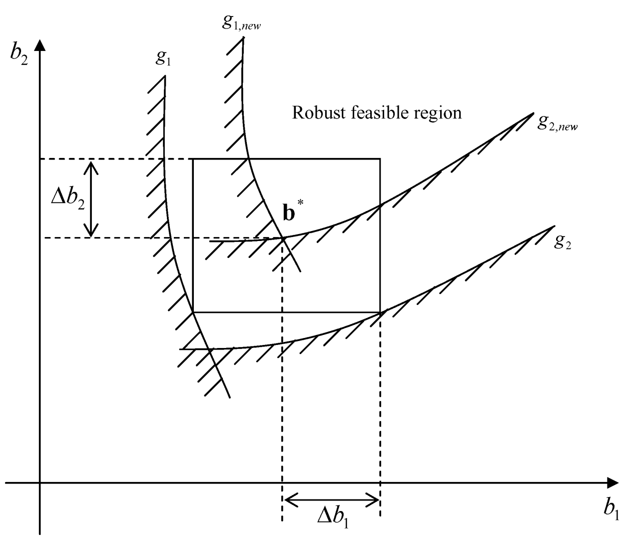

Fig. 6 Change of feasible region of constraints.

design problem, we can hardly separate the mean and variance of design variables. Optimization techniques can obtain design variables by considering the mean and the variance simultaneously. The goal of robust optimization is to solve these problems.

Robust optimization is formulated as Eqs. (8–11). The functions F and G are defined to enhance robustness, and the robustness, of functions is defined as follows: 1) Robustness of the objective function[12,58] can be achieved by reducing the change of the objective with respect to the changes of tolerances for the design variables. Figure 5 presents an objective function that has a minimum at the peak point A, where the objective function can drastically change by small variations of the design variables. Thus, the local minimum point B or C is better for an insensitive design. 2) Robustness of constraints[12,16–19] means that all of the constraints are satisfied within the range of tolerances for the design variables, that is, the feasible region is reduced as shown in Fig. 6. Probabilistic consideration of the objective function and constraints is known as reliability optimization[59] or stochastic optimization.[60–62] They will be briefly mentioned later because we mainly focus on the review of robust design.

Now let us consider the formulation of robust optimization as follows: Find

$$\boldsymbol{b} \in R^n \tag{22}$$

to minimize

$$[\mu_f(\boldsymbol{b},\boldsymbol{p}) \quad \sigma_f(\boldsymbol{b},\boldsymbol{p})] \tag{23}$$

subject to

$$g_j(\boldsymbol{b}+\boldsymbol{z}^b, \boldsymbol{p}+\boldsymbol{z}^p) \leq 0 \, j = 1,\ldots,r \tag{24}$$

$$\boldsymbol{b}_L \leq \boldsymbol{b} \leq \boldsymbol{b}_U \tag{25}$$

Note that the objective function in Eq. (23) is defined in terms of the mean and the standard deviation, that is, the variance. Equation (24) means that all of the constraints should be satisfied despite the noises in design variables and parameters. Therefore, robust optimization methods have been efficiently developed to solve the problem defined as Eqs. (22–25).[11,15,58,63–65] Optimization with uncertainties is similarly defined as Eqs. (22–25).

B. Robustness of Objective Function

Robustness of the objective function has been emphasized more than that of constraints because an insensitive design of the objective function is pursued. The mean μ_f and the variance σ_f of the objective function are approximated as follows:

$$\mu_f \cong f(\boldsymbol{\mu}_b, \boldsymbol{\mu}_p) \tag{26}$$

$$\sigma_f^2 \cong \sum_{i=1}^{n} \left(\frac{\partial f}{\partial b_i}\right)^2 \sigma_{b_i}^2 + \sum_{i=1}^{0} \left(\frac{\partial f}{\partial p_i}\right)^2 \sigma_{p_i}^2 \tag{27}$$

If $\boldsymbol{b}$ and $\boldsymbol{p}$ are assumed to have normal distribution, the mean μ_f of Eq. (26) can use the value of Eq. (2) during the optimization process. Note that Eqs. (26) and (27) are meaningful when the interactions of the design variables and the design parameters are negligible and deviations of them are small.[10,66]

The variance is made of the sensitivity, that is, first derivative, as shown in Eq. (27). If the objective function includes the sensitivity, second-order derivatives of the objective are required in the optimization process. However, calculation of the second-order derivatives of the objective function is so expensive that it is not recommended. Because the constraints of Eq. (24) are represented in terms of the variance, second-order derivatives of the objective are also needed. Thus, robust optimization without second-order derivatives is recommended.

In the simplest method, the objective function is defined by the sensitivity and the second derivatives are directly evaluated. This method is quite expensive but reliable. It has some limits in that the solution can include local information. Belegundu[14] obtained robustness by minimizing the sensitivity information. Jung and Lee[67,68] have shown that robust optimization can be achieved for mass minimization because the second derivative can be easily calculated for the mass of a structure. However, robustness of mass may not be important in engineering design. Han and Kwak[69] have directly used the second-order sensitivity for an objective function that is the natural frequency of a structure. As computer power increases, calculation of the second-order sensitivity is viable for mathematical or small-scale engineering problems, but this is difficult to apply to large-scale problems.

Many methods have been proposed to avoid second-order sensitivity calculation. Sundaresan et al.[15,65] proposed the sensitivity index (SI) to incorporate the evaluation of constraints with the worst combination of design variables in an orthogonal array. Their method can reduce the computing cost. Su and Renaud[64] evaluated the first-order sensitivity of SI automatically. Also, the objective function can be approximated as an explicit regression model. Chen et al.[56] have shown that the second-order sensitivity of the objective function can easily be evaluated if the objective function is approximated by the response surface method (RSM).[70] Also approximation of the objective function using the RSM has been often used in robust optimization. However, the RSM has the risk to converge to a sensitive design because the sensitive part of the objective function can be ignored in the approximation process. The Taguchi method can be applied to robust optimization as explained in Sec. III. The mean and the variance are evaluated by using an outer orthogonal array and minimized by the one-way table. This method does not need sensitivity information.[32,71,72]

Robust optimization has also been developed with uncertainties. Du and Chen proposed the propagating model uncertainty by applying the extreme condition approach and the statistical approach.[73] To integrate robust design with multidisciplinary design optimization, efficient uncertainty analysis has been developed from a computational viewpoint. The system uncertainty analysis and the concurrent subsystem uncertainty analysis methods are developed to improve the computational efficiency in highly coupled analyses. Then the effort for the system level is significantly reduced.[74] Putko et al. considered the geometric uncertainty and flow parameter uncertainty for robust design using the quasi-one-dimensional Euler computational fluid dynamics code.[75,76]

C. Robustness of Constraints

As mentioned earlier, the constraints can be transformed into Eq. (24) in the case when noises exist. Treatment of constraints varies according to the detailed definition of Eq. (24). The research trends are as follows.

1. *Reduction of the Feasible Region*

The feasible region is reduced by noises as shown in Fig. 6. In other words, the constraints in Eq. (27) must be satisfied even though we have z^b and z^p. Thus, tolerance design methods have been applied to robust optimization.

With the noises of constraints, Eq. (24) can be written as[16,60]

$$g_{j,\text{new}} \equiv g_j + k\sigma_{g_j} \leq 0 \tag{28}$$

where k is a user-defined constant depending on the design purpose and σ_{g_j} denotes the standard deviation of the constraint g_j that can be approximated from Eq. (4) as follows[10,60]:

$$\sigma_{g_j}^2 \cong \sum_{i=1}^{n} \left(\frac{\partial g_j}{\partial b_i}\right)^2 \sigma_{b_i}^2 + \sum_{i=1}^{0} \left(\frac{\partial g_j}{\partial p_i}\right)^2 \sigma_{p_i}^2 \tag{29}$$

If the noise is given by a range, the worst case of $g_{j,\text{new}}$ and Eq. (25) becomes

$$g_{j,\text{new}} = g_j + k_j^b \sum_{i=1}^{n} \left|\frac{\partial g_j}{\partial b_i}\right| \left|z_i^b\right| + k_j^p \sum_{i=1}^{0} \left|\frac{\partial g_j}{\partial p_i}\right| \left|z_i^p\right| \tag{30}$$

$$\boldsymbol{b}_L \leq \boldsymbol{b} + \boldsymbol{z}^b \leq \boldsymbol{b}_U \tag{31}$$

where $|z_i^b|$ and $|z_i^p|$ denote the maximum values of the tolerance ranges and k_j^b and k_j^p are user-defined constants depending on the design purpose. Equations (29) and (30) have the first derivatives of constraints. Thus, the second-order derivatives of constraints are needed if mathematical programming is employed as an optimizer. It is similar to the case of the objective function.

The design point $\boldsymbol{b}^*$ of Fig. 6 is the optimum point when the design variables have the tolerances whose ranges are box-type domains as shown in Fig. 6. Here $g_{j,\text{new}}$ is defined to satisfy the constraints within the ranges as follows:

$$g_{j,\text{new}} = \text{maximum}\{g_j(\boldsymbol{b}), \forall \boldsymbol{b} \in \text{the hypersurface of Fig. 6}\} \tag{32}$$

In addition to the methods mentioned, $g_{j,\text{new}}$ can be defined based on the theory of tolerance design, but the concept is very similar.

2. *Probabilistic Analysis*

Probabilistic analysis is adopted from the theories of reliability optimization. It is also being used in the design that considers uncertainties. Satisfaction of constraints can be written as probabilistic forms for normal distribution of noise[59,60,66,67]:

$$P[g_j(\boldsymbol{b} + \boldsymbol{z}^b, \boldsymbol{p} + \boldsymbol{z}^p) \leq \boldsymbol{0}] \geq P_{j,0}, \qquad j = 1, \ldots, r \tag{33}$$

where $P[\cdot]$ denotes probability such that $P_{j,0}$ represents the probability of satisfaction for constraints, whereas the noise $\boldsymbol{z}^b$ or $\boldsymbol{z}^p$ is of normal distribution. If probabilistic distributions of noises are given, Eq. (33) can be rewritten as follows:

$$p[g_j(\boldsymbol{b} + \boldsymbol{z}^b, \boldsymbol{p} + \boldsymbol{z}^p) \leq \boldsymbol{0}] = \int_{g_j(b+z^b,p+z^p)\leq 0} \mathrm{d}(\boldsymbol{z}^b, \boldsymbol{z}^p)\, \mathrm{d}\boldsymbol{z}^b\, \mathrm{d}\boldsymbol{z}^p, \qquad j = 1, \ldots, r \tag{34}$$

where $\mathrm{d}(\boldsymbol{z}^b, \boldsymbol{z}^p)$ is the joint probability density function of probabilistic variables $\boldsymbol{z}^b$ and $\boldsymbol{z}^p$. Evaluation of Eq. (34) is very difficult; therefore, an approximate equation can be used to reduce the computing cost.

To approximate Eq. (34), the moment matching formulation can be used.[77,78] In the moment-matching method, the probability is simplified by assuming that the constraints have normal distribution. It is one of the most popular methods. If the constraints have normal distribution, Eq. (33) becomes

$$\int_{-\infty}^{-g_j(b,p)/\sigma_{g_j}} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-\theta^2}{2}\right) \mathrm{d}\theta \geq \int_{-\infty}^{\phi_j(p_j)} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-t^2}{2}\right) \mathrm{d}t \tag{35}$$

where $\phi_j(p_j)$ is the variable for the probability p_j of the normal distribution and is given as

$$\phi_j(p_j) = \left[g_j(\boldsymbol{b} + \boldsymbol{z}^b, \boldsymbol{p} + \boldsymbol{z}^p) - g_j(\boldsymbol{b}, \boldsymbol{z})\right] / \sigma_{g_j} \tag{36}$$

From the inequality of Eq. (35), Eq. (33) can be modified as

$$g_{j,\text{new}} \equiv g_j + \phi_j(p_j)\sigma_{g_j} \leq 0 \tag{37}$$

Equation (37) provides the base for Eq. (28). Equation (37) can be employed in robust optimization; however, Eq. (28) is used more often.

D. Optimization Methods

The optimization scenario is the key point of robust optimization. Robust optimization has employed nonlinear programming, but second-order derivatives are needed because of the variance in the objective function. Although the second-order derivatives are quite expensive, we may still use nonlinear programming with second-order derivatives. However, some methods have been developed to alleviate the use of the second-order derivatives.[31,60,71,72] Optimization methods have been employed to accommodate the multiobjective function. The multiobjective function can deal with uncertainties.[79,80] Here some multiobjective methods used for robust optimization are introduced.

1. *Weighted Sum Method*

In Sec. IV.C, we considered the robustness of constraints during robust optimization where a feasible region is reduced. As shown in Eq. (23), there are multiple objective functions in robust optimization. Because general optimization considers one objective fnction, various approaches are introduced to define a single objective function from multiobjective functions. The most common approach in multiobjective optimization is the weighted sum method. Equation (23) can be rewritten as[12,31,81,82]

$$f_{\text{new}} = \alpha\left[\mu_f(\boldsymbol{b}, \boldsymbol{p}) / \mu_f^*\right] + (1 - \alpha)\left[\sigma_f(\boldsymbol{b}, \boldsymbol{p}) / \sigma_f^*\right] \tag{38}$$

where μ_f^* and σ_f^* are base values for the mean μ and standard deviation σ of the objective function, respectively. They are used for normalization and usually have the starting values of the optimization process. The weighted sum method is inefficient, but it is popular due to simple and easy application.

2. *Compromise Decision Support Problem*

One type of multiobjective optimization method is goal programming. Designers determine the design goal, transform the goal into goal constraints, and then optimize them with real constraints by means of nonlinear programming. A goal constraint is another expression that an objective function must achieve the design goal. The compromise decision support problem is a modified version of goal programming for robust optimization.[47,56,58,83–85] A compromise decision support problem is a hybrid multiobjective optimization technique that incorporates both traditional mathematical programming and goal programming. The system constraints must be retained without compromise while objectives are properly compromised.

The objective function in a compromised decision support problem is a deviation function $x(d_i^-, d_i^+)$ in terms of deviation variables. Therefore, a robust optimum can be found by minimizing the deviation variables while the constraints are satisfied. The compromise decision support problem with t multiobjective functions is formulated as follows: Find

$$\boldsymbol{b} \in R^n \tag{39}$$

to minimize

$$x = \left\{w_1 h_1\left(d_1^-, d_1^+\right), w_2 h_2\left(d_2^-, d_2^+\right), \ldots, w_t h_t\left(d_t^-, d_t^+\right)\right\} \tag{40}$$

subject to

$$g_i(\boldsymbol{b}, \boldsymbol{p}) < 0, \qquad i = 1, \ldots, r \tag{41}$$

$$f_j(\boldsymbol{b}, \boldsymbol{p}) + d_j^+ - d_j^- = g_j^f \tag{42}$$

$$d_j^- \cdot d_j^+ = 0 \tag{43}$$

$$d_j^- \geq 0, d_j^+ \geq 0, \qquad j = 1, \ldots, t \tag{44}$$

where w_j is the weight and g_j^f is the goal that the jth objective function achieves. Here d_j^- and d_j^+ are the deviation variables that represent the distance between the target level and the actual attainment of the goal and indicate the degrees of shortage and overachievement of the goals, respectively. As a result, we can achieve the goal as much as possible by means of minimizing these deviations. In contrast to traditional goal programming, the compromise decision support program uses real design constraints as shown in Eqs. (39–44).

Various deviation functions are defined in a compromise decision support program. The most general and simplest form is the weighted sum of deviations according to the level of requirement to achieve each goal.[84] The distance function considering the weighted deviation vector has been used as a deviation function.[85]

When a compromise decision support program is used in robust optimization, both the robustness and feasibility of constraints are satisfied and various design goals are achieved by compromising goals. Moreover, deviation variables provide the degrees of shortage and overachievement of the goals. This method can obtain a Pareto solution for a nonconvex problem (see Ref. 85).

3. *Physical Programming*

Messac,[86] Messac and Wilson,[87] and Messac and Ismail-Yahaya[88] have proposed an effective multiobjective optimization method. It is called physical programming. It classifies objectives and includes them in the constraint set according to preference. The preference is expressed by a utility function called an aggregate preference function. For instance, the weighted objective function in Eq. (38) is an example. This approach has been used in robust design with a multiobjective function.[89]

In addition to the described methods, the difference between the maximum and minimum of the objective function has been minimized to achieve the robustness of an objective function.[18] The relationship between robustness and optimum sensitivity has been studied.[17,90,91] Chen et al. have studied the role of robustness in concurrent engineering to enhance productivity.[91] In robust design, a unified method has not yet been proposed, and large-scale problems rarely have been solved. Many practical case studies are needed.

V. Robust Design in Axiomatic Design

Axiomatic design has been proposed by Suh,[2,20] and it provides a framework for robust design. The basic postulate of axiomatic design is that there are two fundamental axioms. The axioms are stated as follows:

1) The independence axiom is to maintain the independence of the functional requirements (FRs).

2) The information axiom is to minimize the information content of the design.

The independence axiom states that the independence of the functional requirements must always be maintained by an appropriate choice of design parameters (DPs). For example, the design equations may be classified into the following three kinds when there are two functional requirements:

$$\begin{Bmatrix} \mathrm{FR}_1 \\ \mathrm{FR}_2 \end{Bmatrix} = \begin{bmatrix} X & 0 \\ 0 & X \end{bmatrix} \begin{Bmatrix} \mathrm{DP}_1 \\ \mathrm{DP}_2 \end{Bmatrix}$$

$$\begin{Bmatrix} \mathrm{FR}_1 \\ \mathrm{FR}_2 \end{Bmatrix} = \begin{bmatrix} X & 0 \\ X & X \end{bmatrix} \begin{Bmatrix} \mathrm{DP}_1 \\ \mathrm{DP}_2 \end{Bmatrix}$$

$$\begin{Bmatrix} \mathrm{FR}_1 \\ \mathrm{FR}_2 \end{Bmatrix} = \begin{bmatrix} X & X \\ X & X \end{bmatrix} \begin{Bmatrix} \mathrm{DP}_1 \\ \mathrm{DP}_2 \end{Bmatrix} \tag{45}$$

In the first case of Eq. (45), the design matrix is diagonal. Each of the functional requirements can be independently satisfied by one DP. This is called an uncoupled design. In the second, the matrix is triangular. The independence of functional requirements can be guaranteed if the DPs are determined in an appropriate sequence. This is called a decoupled design. The third has a full design matrix and results in a coupled design. It should be avoided because any choice of DPs will not satisfy the independence axiom.

The information axiom provides a metric of selecting the best design when there are multiple ones that satisfy the independence axiom. The best value for selected DPs can be obtained by minimizing information content. Information content I is defined in terms of the probability p of satisfying a given functional requirement as

$$I = \log_2(1/p) \tag{46}$$

Information content is defined in terms of the probability of success, which is calculated by specifying the design range and the system range. The design range and the system range of a design with one functional requirement are shown in Fig. 7. The information is defined as

$$I = A_{\mathrm{cr}}/A_{\mathrm{sr}} \tag{47}$$

where A_{cr} is the common range and A_{sr} is the system range. To minimize the information content, 1) the variance of the system must be small and 2) the bias must be eliminated to make the system range within the design range. This policy is similar to the Taguchi method. That is, the minimization of the information content is similar to the robust design concept.

Robust design with axiomatic design is conducted after the independence axiom is satisfied. Information of each FR is minimized. Therefore, it is one FR-one DP problem as follows[20]:

$$\mathrm{FR} = A \cdot \mathrm{DP} \tag{48}$$

Suppose that there are two candidate DPs in Eq. (48) and that the relationships between the FR and the DP are shown in Fig. 8. The gradient of a design is called the stiffness of the design. The specified tolerance of FR can be more easily achieved by lowering

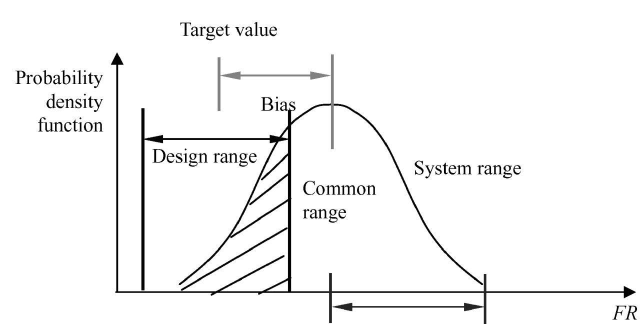

Fig. 7 Information content based on probability density function.

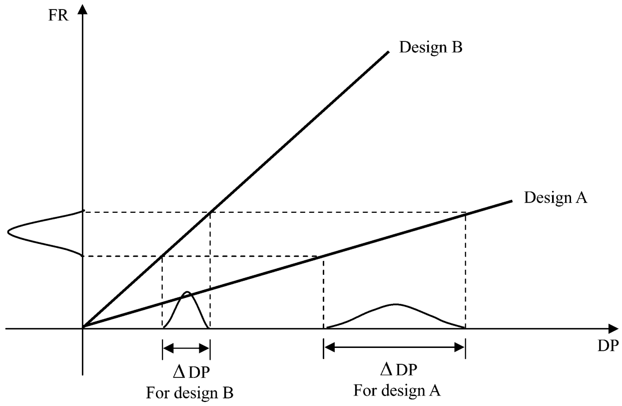

Fig. 8 Distribution of an FR from the two designs.

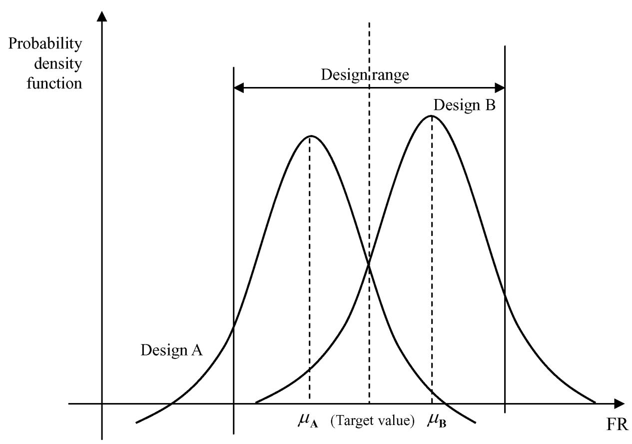

Fig. 9 Distribution of the two designs with target value and design range.

the stiffness of the system (design A). Allowing ΔDP to be larger leads to a robust design. Therefore, the process of robust design[20] is stated as 1) select the design that has a low stiffness to minimize the variation of the system range and 2) minimize the bias between the target and the mean from the selected design.

As mentioned earlier, common aspects exist between the Taguchi method and axiomatic design in that a robust design is induced.[1,8,9] However, different characteristics are discovered as well. The Taguchi method emphasizes how each parameter is determined after the conceptual design. That is, the value of each parameter is determined from the S/N ratio of each row in an orthogonal array. Axiomatic design emphasizes which parameters are chosen in the conceptual stage because a choice of the DP is important as shown in Fig. 8. In other words, selection of a part out of multiple candidates is determined by axiomatic design.

Robust design in axiomatic design is obtained by minimizing the information content. Various methods have been developed for calculation of the information content for robust design.[2,20–22,92–94] In general, there is little interest in the information content of a coupled design because a coupled one basically means a bad design. On the other hand, the calculation of an uncoupled design is very simple. The total information content of an uncoupled design is the sum of the information content of each functional requirement. To calculate the information content of a decoupled design, there are several approaches because the calculation is not simple. Suh[20] used the conditional probability to calculate the information content of a decoupled design. Oh (see Ref. 20) developed a graphical approach when there are two functional requirements. Meanwhile, a generalized method[95–97] was developed by using the multi-integral in the FR or DP domain. This method may not be used frequently because the calculation is very complicated.

Some researchers have been interested in the relationship between axiomatic design and the Taguchi method. Bras and Mistree[98] studied how to use the decision models to perform axiomatic design and robust design using the compromise decision support problem. Park has mentioned the differences and common aspects between the two methods.[97] As shown in Fig. 2, design E is better than design D from the viewpoint of axiomatic design, but design D is better in the viewpoint of the Taguchi concept because the Taguchi method does not use a design range. From this study, Hwang and Park,[21] Hwang,[22] and Park[97] developed the robustness index by using the design range and the probability density function. If the mean values are inside the design range and the distributions are the normal ones as shown in Fig. 9, the two methods select the same design candidate because the information content and the loss function have the same property in the robust design process. Consequently, the Taguchi method and the information axiom of axiomatic design can come to the same conclusion if the independence axiom is satisfied.

VI. Summary

Classification of robust design and its characteristics are surveyed. As mentioned earlier, robust design has been developed in three ways. Because there are numerous case studies for each method, the case studies are not introduced in this paper. Basic ideas and development procedures of each method are only introduced and explained.

Robust design is an excellent concept in that an insensitive or a tolerant design can be obtained. Methods for robust design have been actively developed with much expectation. The methods are established although they are neither perfect nor universal. Robust design has been addressed from the area of design of experiments (DOE) where the Taguchi method can be easily applied. Thus, most of the case studies are from DOE. Design engineers adopted the concept in mechanical design. They developed theories, as introduced in this paper. Many problems have been solved; however, most of them are local or small-scale problems. Especially, examples from astronautics and aeronautics have those characteristics. It is expected that examples from the practical community will be defined. Also, the benefits of robust design should be demonstrated from the examples.

Three methods are introduced in this paper. The characteristics and future direction of each method are summarized as follows:

1) The Taguchi method is quite excellent for DOE, and designers are making efforts to adopt the robustness concept of the method in general design. However, it is not easy to find a scale factor that is exploited in the original Taguchi method. Also, direct use of the S/N ratio is difficult. Although many modified methods have been developed according to the characteristics of application, a rigorous mathematical method for design has not yet been found. Some hybrid methods with robust optimization are expected to solve engineering design problems.

2) Robust optimization is an effort to adopt optimization, which is mathematically excellent, to achieve robustness. However, it was found that a multiobjective function exists and second-order derivatives are needed. Therefore, compromised solutions are pursued, or approximation methods are employed. In this case, questions may arise if we have to use mathematical optimization. Reliability optimization and optimization with uncertainty have similar concepts to that of robust optimization. However, what they pursue is somewhat different. They can also be joined together. For example, the uncertainty concept can be used in the objective function and the reliability concept can be employed for the constraints of robust design. These days, application of the RSM is being actively developed. The RSM has been used for approximation in robust design. The RSM can be further investigated for robust design.

3) In axiomatic design, application of the information axiom can achieve robust design. When the information content is minimized, robust design is achieved and the loss function of the Taguchi method is also minimized. Because axiomatic design is relatively new, there are not many case studies. Also, the case studies mostly use the independence axiom. Therefore, more research is needed for the information axiom to achieve robust design. Robust optimization might be exploited in the decision making process in the application of the information axiom.

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